AIAA JOURNAL
Vol. 31, No. 4, April 1993

Base Pressure of a Sudden Expansion from a Conical Converging Nozzle

Chi-bok Hwang*
Joint Chiefs of Staff, Seoul 3KA, Republic of Korea
and
Wen L. Chow† and Davood Moslemian‡
Florida Atlantic University, Boca Raton, Florida 33431

This investigation concerns the determination of the back-pressure-independent base pressure related to the conical convergent nozzle flow with a sudden enlargement in cross-sectional area. It is recognized at the outset that the problem belongs to the category of strong interaction, where inviscid and viscous flows must be considered together before a solution can be established. The viscous flow analyses, based on the integral formulations, are guided more or less by the boundary-layer concept. The inviscid flowfield is established from the hodograph transformation and the method of characteristics. The point of reattachment behaves as a saddle point singularity for the system of equations describing the viscous flow recompression process. In conjunction with an overall momentum balance, the base pressure and the location within the wake region where recompression starts can be determined. Experimental studies of sudden expansion from conical converging nozzles with specific conical angles and area ratios are also conducted in the laboratory. The results obtained from the theoretical analysis agree fairly well with the experimental data. These results lead to the conclusion that the method developed in this investigation is effective in dealing with problems of this type.

I. Introduction

BASE pressure is one of the important and complicated problems in fluid dynamics. It has been the subject of intensive study for many years because of its academic interest as well as its practical applications. Most of the efforts in this field have been focused on transonic or supersonic external flow past blunt based bodies.[2-7] These efforts have been properly reviewed.[8-10]

Base pressure problems of internal flows are also important in practical applications. The base pressure will affect the performance of an ejector nozzle when operated without the secondary flow. Although the internal base pressure has also been examined extensively for parallel approaching uniform flows, it is believed that the base pressure problem associated with a conical convergent primary nozzle has not been properly studied theoretically. This is the basic motivation behind this investigation.

Perhaps it may be argued that the present problem can be handled by large-scale numerical computations. Because the viscous effects through the conical convergent nozzle immediately upstream of a sudden expansion are very small and can be ignored, it is believed that this flowfield can be effectively established through the technique of hodograph transformation. Indeed, this belief is also the reason why this project was undertaken.

The objective of this research is the investigation on the base pressure ratio P_b/P_o associated with a suddenly expanded internal flow issuing out from a conical convergent nozzle. The geometrical configuration of this study is depicted in Fig. 1. A uniform flow with a small velocity V_a approaches the conical convergent nozzle with an arbitrary angle α. Because of the upstream high stagnation pressure P_o, the flow expands into the downstream cylindrical enlarged duct resulting in a wake region of uniform pressure P_b. It should be emphasized that for the flow system depicted in Fig. 1, the dimensionless base pressure P_b/P_o would normally be dependent upon the ambient pressure ratio P_{amb}/P_o. However, when P_o is high enough that the flow downstream of the enlargement becomes supersonic, this base pressure ratio P_b/P_o will no longer be influenced by the ambient pressure ratio P_{amb}/P_o. This base pressure ratio and the related flowfield are the subjects of the present investigation. The existence of such a flow regime has been observed in the laboratory. It is well known that this base pressure ratio P_b/P_o is strongly dependent on the area ratio A_d/A_c, and the conical nozzle angle α. Although the Reynolds number (Re) would also influence the results, it is known, however, that the Re has a very minor influence on the base pressure ratio as long as it is very large, and the asymptotic invariance of the turbulent flowfield prevails. The finite velocity of approach V_a will also have some influence on the base pressure ratio, but its effect is relatively small as long as R_a/R_c in Fig. 1 is much larger than unity (e.g., $R_a/R_c \geq 4$).

Theoretical analysis on the conical convergent nozzle-free jet flow and the viscous flows based on the integral formulations are presented in the following sections. Finally, results obtained from the computations are presented and compared with the corresponding experimental data.

II. Theoretical Analysis

In many engineering flow problems, the Re is usually very high, and the boundary-layer concept can be applied. This approach can even be applied to "strong interaction" prob-

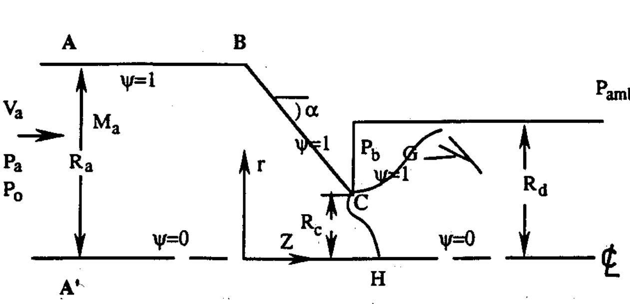

Fig. 1 Configuration of a convergent conical nozzle-free jet flow through a sudden enlargement in cross-sectional area.

Received Dec. 6, 1991; revision received Aug. 17, 1992; accepted for publication Aug. 19, 1992.

*Research Scientist, Department of Weapon Systems.
†Professor of Mechanical Engineering. Associate Fellow AIAA.
‡Associate Professor of Mechanical Engineering.

lems where both viscous and inviscid flow mechanisms must be considered simultaneously before the solution can be established. Indeed the present investigation involves the separation of flow at the corner of the lip of the conical convergent nozzle (Fig. 1) and belongs to the category of strong interaction. The analyses on the inviscid flowfield associated with the conical convergent nozzle-free jet and the viscous flow of turbulent jet mixing and reattachment are separately presented in the following sections.

A. Inviscid Flow Analysis

The establishment of the nozzle-free jet flow and the sonic line is a basic step for the present problem. Because the base pressure is very low in comparison with the upstream stagnation pressure, a curved sonic line prevails within the jet flow. The configuration of the sonic line changes as the back pressure (base pressure) varies. If the growth of the boundary layer along the nozzle wall can be ignored, this inviscid flowfield can be established by the method of hodograph transformation.

From the conventional definition of potential flow in the system of axisymmetric coordinates z, r, it can be shown[1,11] that the stream function ψ can be described by

$$V^2\psi_{vv} + V(M^2+1)\psi_v + \frac{(1-M^2)}{\alpha^2}\psi_{\theta\theta} = \frac{1}{\alpha\sin(\alpha\theta)}\frac{\partial S}{\partial\theta} \tag{1}$$

$$S = \frac{[V^2\psi_v^2 + (1-M^2)\psi_\theta^2/\alpha^2]\sin^2(\alpha\theta)}{2yV\rho(R_c^2V^*/\psi_o) + [\sin(\alpha\theta)/\alpha]\psi_\theta} \tag{2}$$

$$y_\theta = \left(\frac{\psi_o}{R_c^2V^*}\right)\frac{1}{\rho}\left[\alpha\sin(\alpha\theta)\psi_v + \frac{\cos(\alpha\theta)}{V}\psi_\theta\right] \tag{3}$$

$$y_v = \left(\frac{\psi_o}{R_c^2V^*}\right)\frac{1}{\rho}\left[\frac{\cos(\alpha\theta)}{V}\psi_v + \frac{(M^2-1)\sin(\alpha\theta)}{\alpha V^2}\psi_\theta + \frac{S}{V^2}\right] \tag{4}$$

$$Z_\theta = \left(\frac{\psi_o}{R_c^2V^*}\right)\frac{1}{\rho}\frac{1}{r}\left[\alpha\cos(\alpha\theta)\psi_v - \frac{\sin(\alpha\theta)}{V}\psi_\theta\right] \tag{5}$$

$$Z_v = \left(\frac{\psi_o}{R_c^2V^*}\right)\frac{1}{\rho}\frac{1}{r}\left[\frac{(M^2-1)\cos(\alpha\theta)}{\alpha V^2}\psi_\theta - \frac{\sin(\alpha\theta)}{V}\psi_v + \frac{S\cot(\alpha\theta)}{V^2}\right] \tag{6}$$

$$y = \frac{r^2}{2} \tag{7}$$

where all the variables (V, ρ, θ, z, r, ψ) have been normalized respectively by sonic velocity V^*, ρ_o, half-conical angle of the convergent nozzle α, R_c, and ψ_o. ψ_o is the unknown rate of discharge per radian from the conical convergent nozzle.

It is to be noted that the right-hand side of Eq. (1) is the only difference between a two-dimensional and an axisymmetric problem. The original motivation in adopting the method of hodograph transformation lies in its ability to reduce the nonlinear partial differential equation into a problem with linear partial differential equation. However, for the axisymmetric case, this advantage is lost as a result of the presence of the nonlinear term on the right-hand side of Eq. (1). Nevertheless, the simplicity in the specification of the boundary condition for the freejet in the hodograph plane and the ease of finding the solution to the nonlinear equation through iterations make this approach still very attractive.

For the region of supercritical flow in Fig. 2, the hodograph equation needs additional modifications. It is well known that

$$\theta = \pm f(v) + \text{const} \tag{8}$$

represents the simple wave characteristics in a two-dimensional supersonic flow. Here $f(v)$ is given by

$$f(v) = -\frac{1}{\alpha}\left(\sqrt{G}\tan^{-1}\sqrt{\frac{V^2-1}{G-V^2}} - \tan^{-1}\sqrt{G}\sqrt{\frac{V^2-1}{G-V^2}}\right)$$

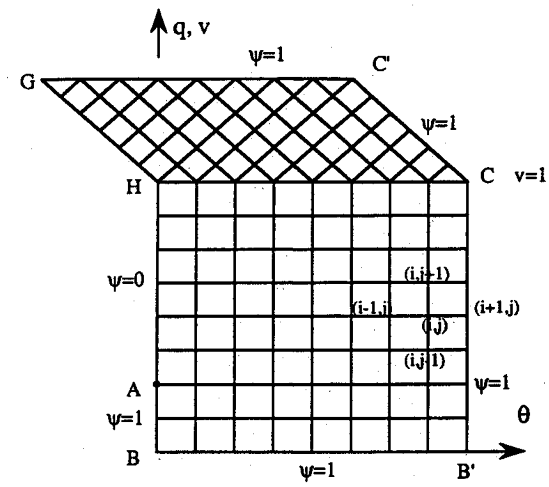

Fig. 2 Computational plane for the nozzle-free jet flow.

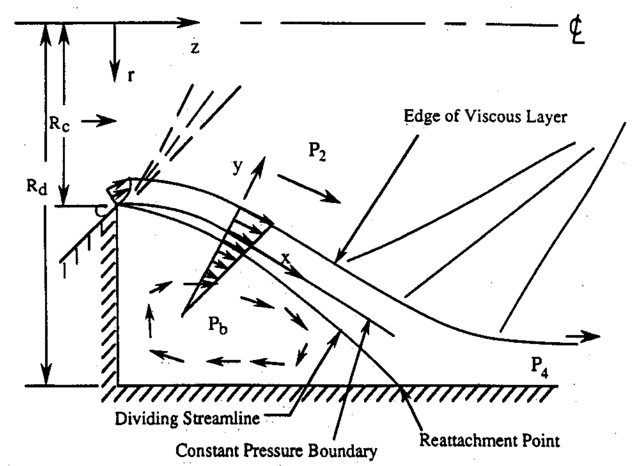

Fig. 3 Suddenly expanded supersonic flow through the convergent conical nozzle.

with $G = \gamma + 1/\gamma - 1$. On defining $q = f(v)$ for this region and substituting the following relations

$$\psi_v = f'(v)\psi_q \tag{9}$$

$$\psi_{vv} = [f'(v)]^2\psi_{qq} + f''(v)\psi_q \tag{10}$$

into Eq. (1), one obtains the hodograph equation for the supersonic flow as

$$\frac{V^2-1}{\alpha}\psi_{qq} - G1\psi_q + \frac{1-V^2}{\alpha}\psi_{\theta\theta} = \frac{1}{\sin(\alpha\theta)}\frac{\partial}{\partial\theta}\left(\frac{S(G-V^2)}{G}\right) \tag{11}$$

with

$$G1 = \frac{2V^4}{(\gamma-1)\sqrt{G(V^2-1)(G-V^2)}}$$

Such a transformation will reduce the curved characteristics (epicycloid) into straight lines in the hodograph plane.

For any point on the sonic line (line CH in Fig. 2), Eq. (1) is simplified into

$$\psi_{vv} + 2\psi_v = \frac{1}{\alpha\sin(\alpha\theta)}\frac{\partial S}{\partial\theta} \tag{12}$$

All the equations are solved through numerical computations. The conventional finite difference forms of Eqs. (1), (11), and (12) can be applied for this purpose.[1]

It is now obvious, from the given boundary conditions in the hodograph plane, that computations are possible by sweeping the hodograph plane from the right-hand side toward the left-hand side. Iterative solutions of ψ, ψ_v, ψ_θ and the configuration of the sonic line can be found through numerical computations. The factor $\psi_o/R_c^2V^*$ is determined from the condition of $y_c = \frac{1}{2}$.

B. Viscous Flow Analysis

Figure 3 is a schematic diagram of a flow past a sudden enlargement in cross-sectional area from a convergent conical nozzle. For an assumed base pressure, a mixing process occurs along the wake boundary, which has been established from the foregoing inviscid analysis. Since the Reynolds number is usually very large, the mixing process is turbulent. The mixing process provides the mechanism of momentum transport and prepares the viscous layer for the subsequent process of the recompression. For the convenience of analysis, it is assumed that the mixing region is divided into two layers along the dividing streamline that separates the flow from the upstream nozzle and the fluid trapped within the wake. As a result of conservation of mass within the wake, the flow above the dividing streamline proceeds downward, and subsequently undergoes a recompression process until the dividing streamline stagnates on the wall. The flow below the dividing streamline is eventually turned back to form the recirculatory wake flow. For simplicity, it is assumed that the isoenergetic flowfield prevails throughout the flow so that consideration of the energy equation is conveniently eliminated.

Processes of Jet Mixing and Recompression

Integral analysis has been employed to describe these processes of jet mixing and recompression. Because this basic approach has been reported for other similar problems,[13-17] a brief description of the analysis is given here.

1) With the given initial boundary layer and a specification of the eddy diffusivity[1] along the dividing streamline, the quasiconstant pressure mixing analysis yields[12] the dimensionless values of the velocity and the shear stress along the dividing streamline, and the thicknesses of the upper and lower viscous layers along the already established inviscid jet boundary.

2) By selection of a location as the starting position of the recompression process, a set of ordinary differential equations can be derived from the continuity and momentum principles to describe the variations of the velocity and the shear stress of the dividing streamline, the thickness of the upper viscous layer, the maximum velocity of the back flow, and the thickness of the lower viscous layer, compatible with the wake geometry. The initial conditions are provided by the upstream mixing analysis. The freestream condition at the edge of the viscous layer is coupled with the inviscid analysis through the method of characteristics. The eddy diffusivity along the dividing streamline during recompression has also been adjusted.

3) It should be emphasized that the normal momentum principle must be employed for the recompression process. It can be shown that the pressure ratio across the viscous layer

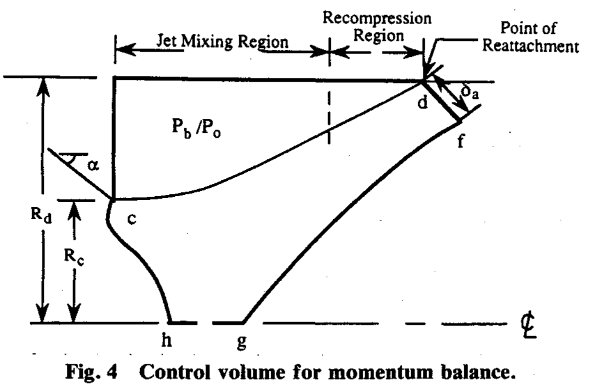

Fig. 4 Control volume for momentum balance.

P_d/P_e is proportional to M_e^2/R, where M_e and R are, respectively, the Mach number and the radius of curvature of the freestream. Since the freestream must turn to the horizontal direction within a fairly short distance, the term $(P_d/P_e - 1)$ can amount to 40%, which is not negligible.

4) The velocity parameter ϕ_d and the slope parameter $\partial\phi/\partial\zeta|_d$ of the dividing streamline in the selected velocity profile has been coupled so as to assure the fact that they vanish together at the point of reattachment.

5) It has been observed again that the point of reattachment behaves as a saddle point singularity of the system of equations describing the process of recompression. That provides the criterion to determine the correct starting location of recompression.

Application of the Momentum Balance

In a previous study of a two-dimensional supersonic external flow problem, the flow redevelopment after the reattachment[18] has been interpreted as a process of relaxation of the pressure difference across the viscous layer. The asymptotic state of this process becomes a saddle point singularity of the system of equations describing the flow, which provides another criterion for establishing the solution of the problem. It was originally hoped that this analysis may be applicable to the present internal flow, even though the pressure at the asymptotic state of the present situation is unknown. However, it was later recognized that for the present problem there is no need to carry out this process of viscous flow redevelopment. With the results of recompression up to the point of reattachment, a momentum balance in the z-direction associated with the control volume as shown in Fig. 4 provides another criterion for the establishment of a solution. It may be seen in this figure that fg is the characteristics passing through f, which is the edge of the viscous layer at the point of reattachment d, and df represents the shear layer there. The curved sonic line is ch and cd is the path of the dividing streamline. This relationship can be given as

$$\text{Residue} = M_{gf} + P_{gf} + M_{df} + P_{df} - M_{hc} - P_{hc} - P_{bs} \quad (13)$$

with

$$M_{gf} = \int_g^f \gamma M^2 \frac{\rho}{\rho^*}\frac{T}{T^*} r \cos\theta(\cos\theta \, dr - \sin\theta \, dz)$$

$$P_{gf} = \int_g^f \frac{P}{P^*} r \, dr$$

$$M_{hc} = \int_h^c \gamma r \cos\theta(\cos\theta \, dr - \sin\theta \, dz)$$

$$P_{hc} = \int_h^c r \, dr = \frac{R_c^2}{2} = \frac{1}{2}$$

$$M_{df} = \gamma M_f^2 \frac{\rho_f}{\rho^*}\frac{T_f}{T^*} \delta_a \cos\theta \int_o^1 \frac{1 - c_f^2}{1 - c_f^2\phi^2}(R_d - \delta_a \cos\theta\zeta) \, d\zeta$$

$$P_{df} = \delta_a^2 \int_o^1 \frac{P}{P^*} \zeta \, d\zeta$$

$$P_{bs} = \frac{P_b}{P^*}\frac{(R_d^2 - R_c^2)}{2}$$

When the correct base pressure is reached, the residue in Eq. (13) should vanish.

III. Method of Calculation

For a given geometrical configuration, P_b/P_o is to be determined. This task can be accomplished through the following step-by-step procedure:

1) Upon selecting a base pressure ratio P_b/P_o (which is usually much lower than P^*/P_o) the inviscid flow through the

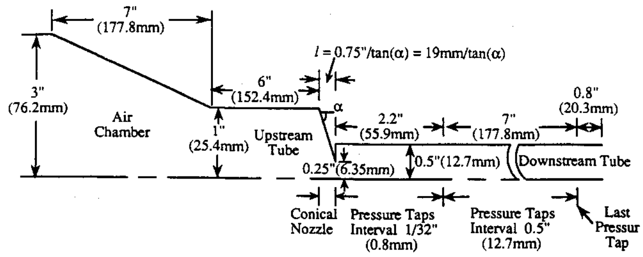

Fig. 5 Physical dimensions of the convergent conical nozzle-free jet test section.

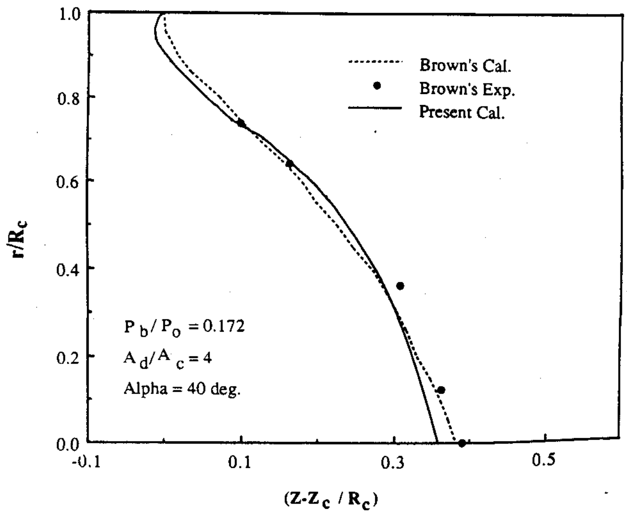

Fig. 6 Comparison of the location of the sonic line with Brown's results.

conical convergent nozzle—including the shape of the sonic line, the approaching flow M_a (or V_a), and the corresponding free jet—in the downstream cylindrical tube can be established numerically.

2) The initial boundary-layer thickness at the lip of the nozzle can be computed by integrating the pair of ordinary differential equations governing the growth of the turbulent boundary layer along the nozzle surface with the established pressure gradient.

3) The quasiconstant pressure turbulent jet mixing can be computed along the wake boundary. The inviscid wake boundary can be established from the method of characteristics for the axisymmetric supersonic flow with the given sonic line configuration.

4) A location is selected along the wake boundary as the beginning position of recompression. The system of nonlinear ordinary differential equations can be integrated to describe recompression. The corresponding adjustment of the condition at the edge of the viscous layer is also described by the method of characteristics for axisymmetric supersonic flow. The saddle point character would provide the answer for the starting location of recompression within the wake region.

5) A momentum balance is applied to the control volume shown in Fig. 4, after the point of reattachment is established. The correct base pressure would yield vanishing residue for the momentum balance.

IV. Experimental Research

Because the present viscous flow analyses are based on an integral approach, only the measurements of the static pressure distributions along the downstream cylindrical tube are necessary to verify the numerical results. The physical dimensions of the test system are presented in Fig. 5. The test section includes a 30-, 45-, or 60-deg convergent conical nozzle, with the same throat diameter of d_c = ½ in. (12.7 mm). For the measurement of wall static pressure as well as the base pressure, a digital multimanometer and a vacuum pressure gauge, both with an accuracy of 1%, are used. An absolute pressure gauge with an accuracy of 1% is used for the measurement of stagnation pressure of the approaching flow. Extensive measurements of the pressure distributions should provide ample evidence on the validity of the analysis for the present problem.

V. Results and Discussion

Before the calculation of the base pressure, the correctness of the location of the sonic line obtained by the present method must be verified. Following the method of calculation outlined in Sec. III, computations were performed for the case with a conical nozzle angle of $\alpha = 40$ deg and the area ratio of $A_d/A_c = 4$. The sonic line obtained by the present method is compared with Brown's[19] theoretical and experimental results in Fig. 6. In general, the location of the sonic line obtained by the present method is reasonably accurate. It should be mentioned that Brown's experiments were not conducted in a rigorous manner, and his analysis was involved in selecting a parameter to match with his data. On the other hand, the present analysis provides the exact solution to the inviscid problem, if the numerical errors of truncation and round-off can be ignored.

The experimental pressure distributions along the downstream tube for nozzle angle of 60 deg with $A_d/A_c = 4$ are presented in Figs. 7 and 8. Figure 7 shows the pressure distributions, P_w/P_o for different levels of stagnation pressures. It

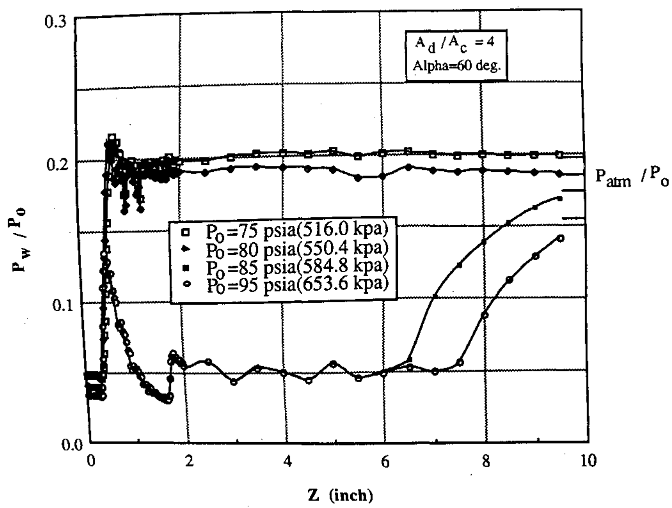

Fig. 7 Experimental wall pressure distributions on the downstream cylindrical tube for various stagnation pressures.

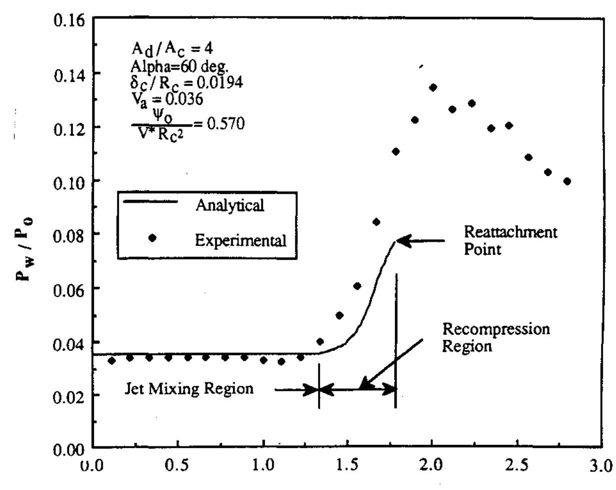

Fig. 8 Comparison between the numerical and experimental wall pressure distributions.

is obvious that the upstream flow pattern is "frozen" for high stagnation pressure ratios P_o/P_{amb} which is the flow regime examined by the present investigation. Indeed, similar phenomena have been observed for conical nozzle angles of 30 and 45 deg. These data are plotted again in Fig. 8 for the purpose of comparison with the results of the present analyses. In general, the base pressure P_b/P_o and the wall pressure distribution P_w/P_o up to the point of reattachment have been predicted fairly well by this analysis. Similar results are also obtained for conical nozzle angles of 30 and 45 deg.

Figure 9 presents the effect of the area ratio A_d/A_c on the base pressure ratio P_b/P_o as predicted by the present analysis for the three nozzle angles. Experimental data for area ratios different from $A_d/A_c = 4$ have been available since 1959[20] and are also shown in Fig. 9 along with the present experimental results. Other than the set of data for $A_d/A_c = 3.5$ reported in Ref. 20, the theoretical analysis is quite adequate for predicting the base pressure associated with conical convergent nozzles. (Concerning Ref. 20, it is believed that the test system at the University of Illinois at that time did not produce the back-pressure-independent base pressure for this area ratio.)

It is interesting to observe that the dividing streamline is rapidly energized immediately after separating from the lip of the nozzle through the jet mixing process, until a maximum value is reached. Subsequently, the velocity of the dividing streamline is reduced as a result of recompression, and the vanishing value is reached at the point of reattachment, even though a continuous transfer of mechanical energy through shearing action takes place throughout this region. This mechanism is also responsible for an additional rise of pressure along the wall (see Fig. 8) after reattachment.[18]

Early expressions for the transport of the fully developed turbulent jet mixing process relied on a similarity parameter σ so that the fully developed velocity profile can be related to a homogeneous coordinate $\eta[\eta = \sigma(y/x)]$ with the corresponding eddy diffusivity given by $\epsilon = 1/4\sigma^2 \, xu_e$, where x is the length along the mixing region. It is empirically known that $\sigma \cong 12$ for incompressible flow and it tends to increase for compressible flow. Since none of the actual situations will correspond to an environment for the fully developed flow, the proper value of σ for any practical situation is not known. In the present investigation, the eddy diffusivity in the recompression region is also directly related to that in the constant pressure region. Thus an effort was directed to see the influence of the similarity parameter σ on the results of the present theoretical analysis. It was found that the variation of the base pressure ratios is very minor even when σ varies from 12 to 18.

It has been realized that due to the difficulty of predicting turbulent transport in detail, it would be advantageous to employ an integral approach so that empirical information

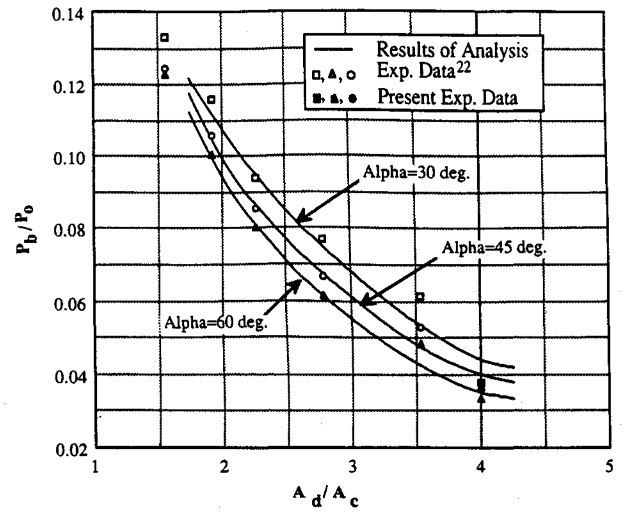

Fig. 9 Variation of the base pressure ratio P_b/P_o as a function of area ratio A_d/A_c for various nozzle angles.

needed to solve the problem can be kept to a minimum. By taking advantage of this integral approach, complicated flow events can be readily described and illustrated by observing important characteristics of the flow. The present problem provides another opportunity for demonstrating the usefulness of the integral approach.

Finally, it should be mentioned that all computations have been carried out with the FAUVAX system (VAX 6320). Although extensive iterations were involved with each specific geometry, only nine minutes of computer time were needed to obtain a solution for the problem.

VI. Conclusions

From the evidence gathered in this series of investigations it may be concluded that

1) The hodograph transformation is effective in describing the inviscid flowfield related to the conical convergent nozzle. The boundary layer on the nozzle surface is thin and its presence would not significantly modify the established flowfield.

2) The integral analyses for the viscous flow are effective in describing the flow events related to the base pressure problem.

3) The method developed in this investigation can adequately predict the flowfield up to the point of reattachment.

4) The momentum balance provides an additional criterion needed to establish the base pressure.

5) From the experimental observation, it is evident that the flowfield downstream of the reattachment-redevelopment is very complex. The only method capable of predicting the flowfield in this region is the large-scale numerical computation. However, correct estimation of dissipation through complicated shock patterns and turbulent mixing within the flow is necessary before accurate simulation of the flow can be achieved.

References

[1]Hwang, C. B., "Base Pressure Resulting from Sudden Expansion in Cross Sectional Area from a Conical Converging Nozzle," Ph.D. Dissertation, Dept. of Mechanical Engineering, Florida Atlantic Univ., Boca Raton, FL, 1991.

[2]Crocco, L., and Lees, L., "A Mixing Theory for the Interaction between Dissipative Flows and Nearly Isentropic Streams," *Journal of the Aeronautical Sciences,* Vol. 19, Oct. 1952, pp. 649–676.

[3]Chapman, D. R., "An Analysis of Base Pressure at Supersonic Velocities and Comparison with Experiment," NACA Rept. 1051, 1951.

[4]Korst, H. H., "A Theory for Base Pressure in Transonic and Supersonic Flow," *Journal of Applied Mechanics,* Vol. 23, Dec. 1956, pp. 593–600.

[5]Lees, L., and Reeves, B. L., "Supersonic Separated and Reattaching Laminar Flows: I. General Theory and Application to Adiabatic Boundary Layer/Shock Wave Interaction," *AIAA Journal,* Vol. 2, No. 11, 1964, pp. 1907–1920.

[6]Nash, J. F., "An Analysis of Two-Dimensional Turbulent Base Flow Including the Effect of the Approaching Boundary Layer," National Physics Lab. Aero. Rept. 1036, Eddington, England, UK, July 1962.

[7]McDonald, H., "An Analysis of the Turbulent Base Pressure Problem in Supersonic Axisymmetric Flow," *Aeronautical Quarterly,* Vol. 16, May 1965, pp. 97–121.

[8]Carpenter, P. W., and Tabakoff, W., "Survey and Evaluation of Supersonic Base Flow Theories," NASA CR-97129, 1968.

[9]Chang, P. K., *Separation of Flow,* Pergamon, London, 1970, pp. 531–607.

[10]Page, R. H., "A Review of Component Analysis of Base Pressure for Supersonic Turbulent Flow," *Proceedings of the Tenth International Symposium on Space Technology and Science,* Tokyo, 1973, pp. 459–469.

[11]Weng, Z. M., Ting, A. L., and Chow, W. L., "Discharge of a Compressible Fluid Through a Control Valve," *Journal of Applied Mechanics,* Vol. 54, No. 4, 1987, pp. 955–960.

[12]Brink, D. F., and Chow, W. L., "Two-dimensional Jet Mixing with a Pressure Gradient," *Journal of Applied Mechanics,* Vol. 42, Series E, No. 1, 1975, pp. 55–60.

[13]Weng, C. H., "Base Pressure Problems Associated with Supersonic Axisymmetric External Flow Configurations," Ph.D. Dissertation, Dept. of Mechanical and Industrial Engineering, Univ. of Illinois at Urbana-Champaign, Urbana, IL, 1975; also *AIAA Journal,* Vol. 16, No. 6, 1978, pp. 553, 554.

[14]Chow, W. L., "Recompression of a Two-dimensional Supersonic Turbulent Free Shear Layer," *Development in Mechanics, Proceedings of the 12th Midwestern Mechanics Conference,* Vol. 6, Univ. of Notre Dame, Notre Dame, IN, Aug. 1971, pp. 319–332.

[15]Liu, J. S. K., "Base Pressure Problems Associated with an Axisymmetric Transonic Flow Past a Backward Facing Step," Ph.D. Dissertation, Dept. of Mechanical and Industrial Engineering, Univ. of Illinois at Urbana-Champaign, Urbana, IL, 1977.

[16]Liu, J. S. K., and Chow, W. L., "Base Pressure Problems Associated with an Axisymmetric Transonic Flow Past a Backward Facing Step," Univ. of Illinois, ME-TR-395-5, Rept. for U.S. Army Grant DAAG29-76-G-0199, Urbana, IL, Nov. 1977, ADA050658; also *AIAA Journal,* Vol. 17, No. 4, 1979, pp. 330, 331.

[17]Chow, W. L., "Base Pressure of a Projectile Within the Transonic Flight Regime," *AIAA Journal,* Vol. 23, No. 3, 1985, pp. 388–395.

[18]Chow, W. L., and Spring, D. J., "Viscous Interaction of Flow Redevelopment after Flow Reattachment with Supersonic External Streams," *AIAA Journal,* Vol. 13, No. 12, 1975, pp. 1576–1584.

[19]Brown, E. F., and Chow, W. L., "Critical Flow Through Convergent Conical Nozzle," *Proceedings of the 1st Symposium on Flow, The Measurement and Control in Science and Industry,* Vol. 1, Instrument Society of America, 1974, pp. 231–240.

[20]Korst, H. H., Chow, W. L., and Zumwalt, G. W., "Research on Transonic and Supersonic Flow of a Real Fluid at Abrupt Increases in Cross Section," Dept. of Mechanical and Industrial Engineering, Univ. of Illinois, ME-TR-392-5, Urbana, IL, 1959.